Thermally Induced Vibrations of a Flexible Appendage Attached to a Spacecraft

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Thermally induced vibrations of a flexible appendage attached to a spacecraft containing a rotor are investigated with particular emphasis on the behavior near resonance. Resonance will occur when the natural frequency of the appendage is approximately equal to twice or half the nutational frequency. In the first case, it is possible for the vibration of the appendage to be unstable through the instrumentality of the parametric excitation. Criteria for the onset of the unstable motion of the appendage are established. In the second case, the vibration of the appendage builds up to a finite value through the resonant exictation. The stationary amplitude of the vibration is obtained. The method of averaging is employed to analyze the phenomena. Finally, the dynamic characteristics of the system near the resonance are investigated numerically.

Introduction

IN the early stages of space exploration, spacecraft tended to be small, mechanically simple, and essentially inflexible. However, in recent years, spacecraft designers have evolved very complex mechanical configurations to meet increasingly demanding missions. Current designs of spacecraft employ large, highly flexible appendages as antennas or solar arrays. The presence of large flexible appendages on a spacecraft necessitates re-examining the basic characteristics of spacecraft dynamics. There is much information in the literature on the effect of energy dissipations in the spacecraft upon the attitude stability of this class of spacecraft.[1-5] On the other hand, several papers have been published to explain anomalous behaviors of this class of spacecraft due to dynamic interactions of the flexible appendages with the environments. Effects of deformations of flexible appendages caused by solar heating, as well as gravitational, magnetic, and aerodynamic effects can be quite significant. The despin mechanism of a spinning spacecraft with flexible appendages due to the action of solar radiation has been investigated by Etkin and Hughes[6] and by Hughes and Charches.[7] Thermal flutter phenomena of long flexible appendages have been analyzed by a number of authors.[8-10] Thermally induced nutational body motions of a spinning spacecraft with flexible appendages have been studied by the present author.[11]

The purpose of this paper is to predict new types of thermally induced vibration of a flexible appendage attached to a spacecraft containing a rotor. The explanation of the phenomena is as follows. Consider a spacecraft composed of a main body and a rotor (Fig. 1). A long flexible appendage is attached on the spacecraft, so as to be parallel to the spin axis of the rotor. Solar radiation is assumed normal to the spin axis. When nutational body motions of the spacecraft exist, the heat input of the appendage caused by solar heating becomes a time periodic function. The periodic heat input produces bending vibrations of the appendage. When the natural frequency of the appendage is approximately equal to twice the nutational frequency, it is possible for the vibration to become unstable through the instrumentality of a parametric excitation. Criteria for the onset of the instability are determined in terms of the ratio of the natural frequency of the appendage to the nutational frequency, the amplitude of the nutational body motion, and certain constants. On the other hand, when the natural frequency of the appendage is approximately equal to half the nutational frequency, the thermal effect of solar heating occurs at a frequency near the natural frequency of the appendage. The vibration , then, gradually builds up, by the external resonance, to a certain value. The stationary amplitude of the vibration of the appendage is obtained. Finally, the dynamic characteristics of the system are investigated numerically for the resonance modes.

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Equations of Motion

A spacecraft, chosen for the analysis, is composed of a rigid main body and a symmetric rotor, which is shown schematically in Fig. 1. A flexible appendage is attached to the main body. A reference frame (X,Y,Z) is fixed in the main body so that the Z axis is taken to be parallel to the spin axis of the rotor, and the origin is coincident with the mass center C of the undeformed total system. The appendage is modeled as a thin rod clamped on the main body at $Z=h$, which is assumed to be flexible in the XZ plane and stiff in the YZ plane. The angular velocity of the reference frame (X,Y,Z) has the components $(\omega_1,\omega_2,\omega_3)$ in the (X,Y,Z) frame. A relative rotational angle of the rotor to the main body is denoted by ϕ. A bending deformation of the appendage is denoted by u. The total kinetic energy T of the system is given by

$$2T=I_1\omega_1^2+I_2\omega_2^2+I_3\omega_3^2+I_R(\dot{\phi}^2+2\omega_3\dot{\phi})$$
$$+\mu\int_h^{h+\ell}\{\dot{u}^2+2\omega_2\dot{u}z+u^2(\omega_2^2+\omega_3^2)-2\omega_1\omega_3uz\}dz \quad (1)$$

where I_1, I_2, I_3 are the moments of inertia of the total system in the undeformed state about X, Y, Z axes, respectively, I_R is the moment of inertia of the rotor about the Z axis, ℓ is the length of the appendage, and μ is the mass per unit length of the appendage. In the preceding derivation, the mass center of the total configuration is assumed to remain fixed at the origin of the frame (X, Y, Z). The elastic potential energy U which arises from the strain energy due to deformations of the appendage is given by

$$2U=B\int_h^{h+\ell}\left(\frac{\partial^2}{\partial z^2}u\right)^2dz \quad (2)$$

where B is the bending stiffness of the appendage. The energy dissipation which results from elastic deformations of the appendage is represented by Rayleigh's dissipation function

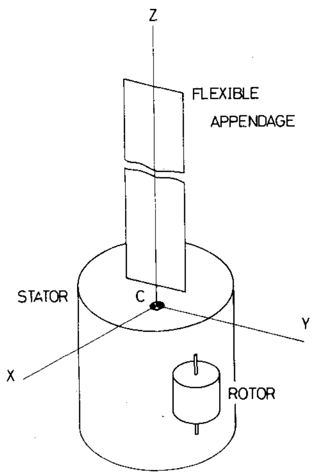

Fig. 1 Spacecraft configuration.

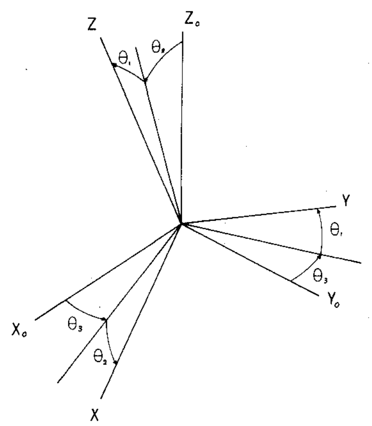

Fig. 2 Euler angle definitions.

F, which is given by

$$F=\mu\gamma\int_h^{h+\ell} \dot{u}^2 \, dz \tag{3}$$

where γ is the damping ratio of the appendage.

The appendage will be deformed by solar radiation induced different heating. The thermal bending moment M_T at any section of the appendage about the Y axis is related to the variation of temperature through the appendage. Now, we shall make the following assumption regarding the thermal bending moment; a thermal bending moment M_T at any section of the appendage about the Y axis is assumed to be governed by the equation

$$(d/dt)M_T+M_T/\tau=K\cos[\alpha+(\partial/\partial z)u] \tag{4}$$

where τ is characteristic time, K is a small thermal bending constant, α is an angle between solar radiation and the normal to the undeformed appendage. This is the same equation as proposed and justified by Etkin and Hughes[6] for describing the heat-transfer process across the boom. The work δW done by the thermal bending moment in an arbitrary displacement δu takes the form

$$\delta W=\int_h^{h+\ell}\left(M_T\frac{\partial^2}{\partial z^2}\delta u\right)dz \tag{5}$$

Solving Eq. (4) and substituting into Eq. (5) we obtain

$$\delta W=\int_h^{h+\ell}\left[\int_0^t e^{-(t-t')/\tau}K\cos\left(\alpha+\frac{\partial}{\partial z}u\right)dt'\right]dz \tag{6}$$

The attitude motion of the spacecraft also is affected by external torques of various forms. These torques are, however, so small that we neglect them in the following analysis. Now, in order to perform a dynamic analysis on the basis of the modal analysis, we represent an elastic deformation by the following series

$$u(z,t)=\sum_{n=1}^{\infty}\ell P_n(t)E_n(\hat{z}) \tag{7}$$

where $E_n(\hat{z})$ are normal modes associated with a cantilever and P_n the expansion coefficients, which are the functions of time, $\hat{z}=z/\ell$. For the functions $E_n(\hat{z})$, we have the equation

$$(\partial^4/\partial\hat{z}^4)E_n(\hat{z})-\chi_n^4E_n(\hat{z})=0 \tag{8}$$

and the boundary conditions

$$E_n(\hat{z})=(\partial/\partial\hat{z})E_n(\hat{z})=0 \quad \text{at} \quad \hat{z}=h/\ell$$

$$(\partial^2/\partial\hat{z}^2)E_n(\hat{z})=(\partial^3/\partial\hat{z}^3)E_n(\hat{z})=0 \quad \text{at} \quad \hat{z}=1+h/\ell$$

where χ_n^4 are the eigenvalues of the normal modes $E_n(\hat{z})$. In addition, they are normalized such that

$$\int_{h/\ell}^{1+h/\ell}E_n(\hat{z})E_m(\hat{z})d\hat{z}=\delta_{n,m} \tag{9}$$

where $\delta_{n,m}$ is Kronecker's delta. In what follows, attention will be paid to the case in which the appendages are excited near the first natural frequency; the nutational frequency is near the first natural frequency of the appendage. In this case, the deflection can be approximated by the first mode, i.e.,

$$u(z,t)=\ell P_1E_1(\hat{z}) \tag{10}$$

and we shall neglect, in the following, the suffix 1.

Next, in order to specify the instantaneous attitude of the spacecraft with respect to the inertia space, we shall introduce an inertia fixed reference frame (X_0, Y_0, Z_0) where the axis X_0 is parallel to solar radiation. The frames (X_0, Y_0, Z_0) and (X, Y, Z) may be related by specifying three Euler angles θ_1, θ_2, θ_3 as follows (Fig. 2). Initially the axes X, Y, and Z are aligned with X_0, Y_0, and Z_0 respectively. Term θ_3 denotes the first right-handed rotation about the Z_0 axis, θ_2 denotes a rotation about the new Y_0 axis, and θ_1 denotes a rotation about the X axis. The angular velocity components ω_1, ω_2, and ω_3 are expressed, using the Euler's angle θ_1, θ_2, and θ_3, as follows

$$\omega_1=\dot{\theta}_1-\dot{\theta}_3\sin\theta_2 \tag{11a}$$

$$\omega_2=\dot{\theta}_2\cos\theta_1+\dot{\theta}_3\sin\theta_1\cos\theta_2 \tag{11b}$$

$$\omega_3=-\dot{\theta}_2\sin\theta_1+\dot{\theta}_3\cos\theta_1\cos\theta_2 \tag{11c}$$

The angle α is given by

$$\alpha=\cos^{-1}(\cos\theta_2\cos\theta_3) \tag{11d}$$

The spacecraft is supposed to be controlled so that solar radiation is in the XZ plane. Besides this condition, we also shall suppose that the order of magnitude of the time during which the X axis turns towards solar radiation is small, i.e., the order of magnitude of the required time is small compared with the time interval of the order of the period of nutational body motions. We therefore can suppose that

$$\theta_3=0 \tag{12}$$

Furthermore, we shall suppose that, in this paper, attitude motions of the spacecraft are small i.e., the variables θ_1, θ_2, and P are small. Equations (1-3 and 6) are, on substituting Eqs. (10-12), and neglecting small quantities of the fourth order, reduced to

$$2T=I_1\dot\theta_1^2+I_2\dot\theta_2^2+\mu\ell^3\dot P^2+I_R(\dot\phi^2-2\dot\phi\dot\theta_2\theta_1)+2\mu\ell^3\dot P\dot\theta_2\langle E\hat z\rangle \tag{13}$$

$$2U=B\chi^4P^2/\ell \tag{14}$$

$$F=\mu\ell^3\gamma\dot P^2 \tag{15}$$

$$\delta W=\delta P\int_o^t e^{-(t-t')/\tau}K[\langle E''\rangle - \tfrac{1}{2}(\langle E''\rangle\theta_2^2+2\langle E'E''\rangle\theta_2 P)]\mathrm{d}t' \tag{16}$$

where

$$\langle f\rangle=\int_{\kappa/\ell}^{1+\kappa/\ell} f d\hat z.$$

The dash denotes the derivative with respect to $\hat z$.

The equations for the generalized coordinates θ_1, θ_2, ϕ, and P are, by Lagrange equation, given in the form

$$(\mathrm{d}/\mathrm{d}t)(\partial T/\partial\dot\theta_1)-(\partial T/\partial\theta_1)=0 \tag{17}$$

$$(\mathrm{d}/\mathrm{d}t)(\partial T/\partial\dot\theta_2)=0 \tag{18}$$

$$(\mathrm{d}/\mathrm{d}t)(\partial T/\partial\dot\phi)=0 \tag{19}$$

$$(\mathrm{d}/\mathrm{d}t)(\partial T/\partial\dot P)+(\partial U/\partial P)=-(\partial F/\partial\dot P)+(\delta W/\delta P) \tag{20}$$

Substituting Eqs. (13-16) in Eqs. (17-20) we obtain for θ_1, θ_2, ϕ, and P the equations

$$I_1\ddot\theta_1+I_R\dot\phi\dot\theta_2=0 \tag{21}$$

$$I_2\ddot\theta_2-I_R(\dot\phi\dot\theta_1+\ddot\phi\theta_1)=-\mu\ell^3\langle E\hat z\rangle\ddot P \tag{22}$$

$$\ddot\phi-(\ddot\theta_2\theta_1+\dot\theta_2\dot\theta_1)=0 \tag{23}$$

$$\mu\ell^3\ddot P+(B\chi^4/\ell)P=-\mu\ell^3\langle E\hat z\rangle\ddot\theta_2-2\mu\ell^3\gamma\dot P+\int_o^t e^{-(t-t')/\tau}K[\langle E''\rangle-\tfrac{1}{2}(\langle E''\rangle\theta_2^2+2\langle E'E''\rangle\theta_2P)]\mathrm{d}t' \tag{24}$$

Eq. (23) easily is solved to obtain

$$\dot\phi-(\dot\theta_2\theta_1)=\Delta_o(\text{a constant}) \tag{25}$$

Substitution of Eq. (25) into Eq. (21), neglecting higher terms, and integrating once leads to

$$I_1\dot\theta_1+I_R\Delta_o\theta_2=\pi_o \quad (\text{a small constant}) \tag{26}$$

Substituting Eqs. (25) and (26) to Eqs. (22) and (24), neglecting higher terms we obtain

$$\ddot\theta_2+k_1^2\theta_2=-(\mu\ell^3/I_2)\langle E\hat z\rangle\ddot P+C_o \tag{27}$$

$$\ddot P+k_2^2P=-2\gamma\dot P-\langle E\hat z\rangle\ddot\theta_2+\int_o^t e^{-(t-t')/\tau}K/(\mu\ell^3)\times[\langle E''\rangle-\tfrac{1}{2}(\langle E''\rangle\theta_2^2+2\langle E'E''\rangle\theta_2P)]\mathrm{d}t' \tag{28}$$

where $k_1^2=(I_R\Delta_o)^2/(I_1\ I_2)$, the nutational frequency, $k_2^2=B\chi^4/(\mu\ell^4)$, the natural frequency of the bending vibration of the appendage, and $C_o=I_R\Delta_o\pi_o/(I_1\ I_2)$, a small constant. The third term in the right-hand side of Eq. (28) represents a nonlinear coupling between the vibration of the appendage and the nutational body motion owing to the solar heating. Now we will investigate this condition. The nonlinear coupling contributes something of particular interest when the following conditions are satisfied

Case 1 $\quad k_1\simeq 2k_2 \quad$ (29a)

Case 2 $\quad k_1\simeq(\tfrac{1}{2})k_2 \quad$ (29b)

Analysis

Let us first study case 1. Introduce new complex variables $\hat a$ and $\hat b$ by the equations

$$\theta_2=\hat a e^{ik_1t}+\hat a^*e^{-ik_1t} \tag{30a}$$

$$P=\hat b e^{ik_1t/2}+\hat b^*e^{-ik_1t/2} \tag{30b}$$

$$\dot{\hat a}e^{ik_1t}+\dot{\hat a}^*e^{-ik_1t}=0 \tag{30c}$$

$$\dot{\hat b}e^{ik_1t/2}+\dot{\hat b}^*e^{-ik_1t/2}=0 \tag{30d}$$

where f^* is the complex conjugate of a complex variable f. Equations (27) and (28) give for $\hat a$ and $\hat b$ the equations,

$$\dot{\hat a}=[1/(2ik_1)]\{(\mu\ell^3\langle E\hat z\rangle/I_2)[(ik_1/2)\times(\hat b e^{ik_1t/2}-\hat b^*e^{-ik_1t/2})-(k_1/2)^2\times(\hat b e^{ik_1t/2}+\hat b^*e^{-ik_1t/2})]+C_o\}e^{-ik_1t} \tag{31}$$

$$\begin{aligned}\dot{\hat b}=(1/(ik_1)](&-\Delta_1(\hat b e^{ik_1t/2}+\hat b^*e^{-ik_1t/2})-\langle E\hat z\rangle\\&\times[ik_1(\hat a e^{ik_1t}-\hat a^*e^{-ik_1t})-k_1^2(\hat a e^{ik_1t}+\hat a^*e^{-ik_1t})\\&-i\gamma k_1(\hat b e^{ik_1t/2}-\hat b^*e^{-ik_1t/2})+(K/\mu\ell^3)\\&\times\int_o^t e^{-(t-t')/\tau}\{\langle E''\rangle-\tfrac{1}{2}[\langle E''\rangle(\hat a e^{ik_1t'}\\&+\hat a^*e^{-ik_1t'})^2+2\langle E'E''\rangle(\hat a e^{ik_1t'}+\hat a^*e^{-ik_1t'})\\&\times(\hat b e^{ik_1t'/2}+\hat b^*e^{-ik_1t'/2})]\}\mathrm{d}t')e^{-ik_1t/2}\end{aligned} \tag{32}$$

where $\Delta_1=k_2^2-(k_1/2)^2$. Since the parameters Δ_1, γ, $\langle E\hat z\rangle$, K, and C_o are small, we can suppose that the right-hand side terms of Eqs. (31), and (32) are small. This means that the terms $\hat a$ and $\hat b$ vary only slowly with time. Hence, the method of averaging[12] gives a satisfactory approximation to these equations. Integrating Eqs. (31) and (32) between the limit t and $t+(4\pi/k_1)$, and considering $\hat a$ and $\hat b$ as remaining constant in this interval, we obtain, as the first approximation, the equations

$$\dot{\hat a}=0 \tag{33a}$$

$$\dot{\hat b}=[(i\Delta_1/k_1)-\gamma]\hat b+iK\langle E'E''\rangle\,\hat a\hat b^*/\times\{\mu\ell^3k_1[(1/\tau)+(ik_1/2)]\} \tag{33b}$$

Equations (33) can be found to obtain the steady state solutions,

$$\hat a=\hat a_o\ (\text{a constant}),\ \hat b=0 \tag{34}$$

Let us now consider the stability of the steady-state solutions, Eqs. (34). Perturbed motions in the neighborhood of the solution (34) may be expressed in the form,

$$\hat a=\hat a_o+\delta\hat a,\ \hat b=\delta\hat b \tag{35}$$

where $\delta\hat{a}$, $\delta\hat{b}$ are perturbations. Introducing Eq. (35) into Eqs. (33) and neglecting higher terms in $\delta\hat{a}$, $\delta\hat{b}$, we obtain the variational equations,

$$\delta\dot{\hat{a}}_r = 0,\ \delta\dot{\hat{a}}_i = 0 \tag{36a}$$

$$\delta\dot{\hat{b}}_r = -(\gamma\delta\hat{b}_r + \Delta_1\delta\hat{b}_i/k_1) + K\langle E'E''\rangle/(k_1\mu\ell^3) \times [(1/\tau)(\hat{a}_{oi}\delta\hat{b}_r - \hat{a}_{or}\delta\hat{b}_i) - (k_1/2)(\hat{a}_{or}\delta\hat{b}_r + \hat{a}_{oi}\delta\hat{b}_i)]/[(i/\tau)^2 + (k_1/2)^2] \tag{36b}$$

$$\delta\dot{\hat{b}}_i = -(\gamma\delta\hat{b}_i - \Delta_1\delta\hat{b}_r/k_1) - K\langle E'E''\rangle/(k_1\mu\ell^3) \times [(1/\tau)(\hat{a}_{or}\delta\hat{b}_r + \hat{a}_i\delta\hat{b}_i) + (k_1/2) \times (\hat{a}_{oi}\delta\hat{b}_r - \hat{a}_{or}\delta\hat{b}_i)]/[(1/\tau)^2 + (k_1/2)^2] \tag{36c}$$

where f_r is the real part of a complex variable f and f_i is the imaginary part of f.

The characteristic equation of Eq. (36) is as follows:

$$s^2 + 2\gamma s + \{\gamma^2 + (\Delta_1/k_1)^2 - [K\langle E'E''\rangle/(k_1\mu\ell^3)]^2 \times (\hat{a}_{or}^2 + \hat{a}_{oi}^2)/[(1/\tau)^2 + (k_1/2)^2]\} = 0 \tag{37}$$

as the stability criterion, the roots of the characteristic equation must not have a positive real part, we have

$$\gamma > 0 \tag{38}$$

$$(\gamma/k_1)^2 + (\Delta_1/k_1^2)^2 - [K\langle E'E''\rangle/(k_1^3\mu\ell^3)]^2 \times (\hat{a}_{or}^2 + \hat{a}_{oi}^2)/[1/(\tau k_1)^2 + 1/2^2] > 0 \tag{39}$$

If the amplitude a of the variable θ_2 is introduced, the condition (39) becomes

$$(\gamma/k_1)^2 + (\Delta_1/k_1^2)^2 - [K\langle E'E''\rangle/ \times (\mu\ell^3 k_1^3)]^2 a_o^2/(\tau k_1)^2 + 1/2^2] > 0 \tag{40}$$

Since we are concerned with a damped mechanical system, the condition (38) always is fulfilled. Let the condition for stability, Eq. (40), be represented in Fig. 3 where the system parameters are given by

$$\gamma/k_1 = 0.05,\ K/(k_1^3\mu\ell^3) = 0.6,\ 1/(\tau k_1) = 0.5 \tag{41}$$

The shaded area is the region of instability, whereas the unshaded part is stable. The condition (40) has a simple physical meaning: if the magnitude of nutational body motions exceeds a certain critical value, vibrations of the appendage absorb the energy, and build up to larger values.

Then, we proceed to study case 2. Introducing new variables $\tilde{a}$ and $\tilde{b}$ by the equations

$$\theta_2 = \tilde{a}e^{ik_1t} + \tilde{a}^*e^{-ik_1t} \tag{42a}$$

$$P = \tilde{b}e^{i2k_1t} + \tilde{b}^*e^{-i2k_1t} \tag{42b}$$

$$\dot{\tilde{a}}e^{ik_1t} + \dot{\tilde{a}}^*e^{-ik_1t} = 0 \tag{42c}$$

$$\dot{\tilde{b}}e^{i2k_1t} + \dot{\tilde{b}}^*e^{-i2k_1t} = 0 \tag{42d}$$

and substituting them into Eqs. (27) and (28), we obtain

$$\dot{\tilde{a}} = [1/(2ik_1)]\{(\mu\ell^3\langle E\hat{z}\rangle/I_2)[(i2k_1)(\dot{\tilde{b}}e^{i2k_1t} - \dot{\tilde{b}}^*e^{-i2k_1t}) - (2k_1)^2(\tilde{b}e^{i2k_1t} + \tilde{b}^*e^{-i2k_1t})] + C_o\}e^{-ik_1t} \tag{43}$$

$$\dot{\tilde{b}} = [1/(4ik_1)](-\Delta_2(\tilde{b}e^{i2k_1t} + \tilde{b}^*e^{-i2k_1t}) - \langle E\hat{z}\rangle[(ik_1) \times (\dot{\tilde{a}}e^{ik_1t} - \dot{\tilde{a}}^*e^{-ik_1t}) - k_1^2(\tilde{a}e^{ik_1t} + \tilde{a}^*e^{-ik_1t})] - i4k_1\gamma(\tilde{b}e^{i2k_1t} - \tilde{b}^*e^{-i2k_1t}) + (K/\mu\ell^3)\int_0^t e^{-(t-t')/\tau}\{\langle E''\rangle - 1/2[\langle E''\rangle(\tilde{a}e^{ik_1t'} + \tilde{a}^*e^{-ik_1t'})^2 + 2\langle E'E''\rangle(\tilde{a}e^{ik_1t'} + \tilde{a}^*e^{-ik_1t'}) (\tilde{b}e^{i2k_1t'} + \tilde{b}^*e^{-i2k_1t'})]\}dt')e^{-i2k_1t} \tag{44}$$

where $\Delta_2 = k_2^2 - (2k_1)^2$. Since the right-hand side terms of Eqs. (43) and (44) are small, we can apply the method of averaging to obtain approximate solutions to Eqs. (43) and (44). Application of the method of averaging to these equations leads to following first approximation equations

$$\dot{\tilde{a}} = 0 \tag{45a}$$

$$\dot{\tilde{b}} = (i\Delta_2/4k_1 - \gamma)\tilde{b} + iK\langle E''\rangle\tilde{a}^2/\times[(8k_1\mu\ell^3)(1/\tau + i2k_1)] \tag{45b}$$

Equations (45) have the steady-state solutions

$$\tilde{a} = \tilde{a}_o \text{ (a constant)} \tag{46a}$$

$$\tilde{b} = iK\langle E''\rangle\tilde{a}_o^2/[(i\Delta_2/4k_1 - \gamma)\times(8k_1\mu\ell^3)(1/\tau + 2ik_1)] \tag{46b}$$

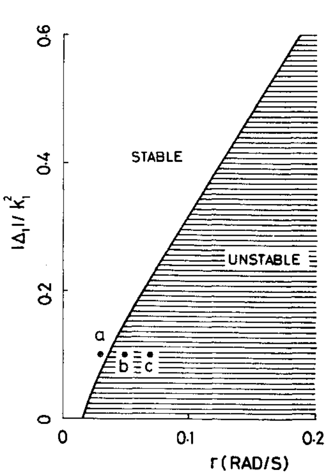

Fig. 3 Stability chart.

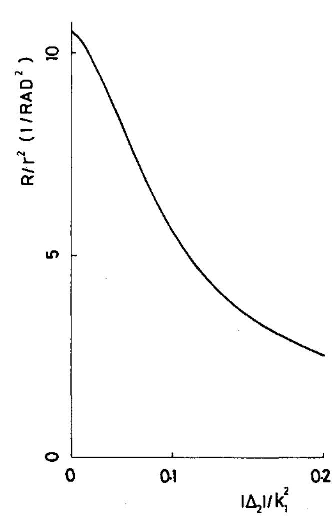

Fig. 4 Amplitude characteristic of the vibration of the appendage.

The amplitude R of the vibration of the appendage is, from Eqs. (46), given by

$$R=K\langle E''\rangle\gamma_o^2/((8k_1^3\mu\ell^3)\{(\Delta_2/4k_1^2)+(\gamma/k_1)^2][(1/\tau k_1)^2+4.0]\}^{1/2}) \quad (47)$$

In this case, the amplitude of the vibration of the appendage builds up to a finite value, Eq. (47), through the external resonance. Figure 4 shows the relationship between Δ_2 and R in the case where

$$\gamma/k_1=0.05, K/[(2k_1)^3\mu\ell^3]=0.12, \quad 1/\tau k_1=0.5 \quad (48)$$

Finally, we shall investigate the dynamic repsonses of the system for the resonance mode numerically on the basis of Eqs. (27) and (28). For case 1, the time-repsonse curves of the system are shown in Fig. 5. The special case we consider is that in which the system parameters are given by

$$\gamma/k_1=0.05, \quad K/(k_1^3\mu\ell^3)=0.06, \quad \Delta_1/k_1^2=0.1 \quad (49)$$

The critical value $a=0.035$ rad., beyond which the vibration of the appendage will become unstable, is predicted by Eq. (41) for this case. Three different initial values are adopted for the amplitude of θ_2, i.e., 0.03 rad. (case a), 0.05 rad. (case b), 0.07 rad. (case c). Case a corresponds to the case where the system starts with an initial condition in the stable region, the unshaded part of Fig. 3. In this case, instead of adding energy by solar heating, energy is withdrawn only by dissipation in the appendage. Hence, the system may be stable. Cases b and c correspond to the cases of the system starting with initial conditions in the unstable region. The solar energy is, in these cases, introduced into the system; vibrations of the appendage absorb the energy and transmit it to nutational body motions of the spacecraft. For the case b, R increases as far as $t\simeq 130$ sec where R reaches a maximum beyond which it decreases rapidly, whereas a is damped in the course of time. The result indicates that, in this case, the withdrawal of energy of the nutational body motion by dissipation in the appendage overcomes the injecting of energy by solar heating. This, in turn, results in the damping of a nutational body motion. On the other hand, a vibration of the appendage absorbs the solar energy and is amplified until the amplitude of nutational body motions reaches the stability curve in Fig. 3; then, a further decrease in the amplitude of nutational body motions results in the damping of a vibration of the appendage. For case c, because the adding of energy into the nutational body motion by solar heating overcomes the withdrawal of energy by the dissipation in the appendage, the amplitude of the nutational body motion as well as the amplitude of vibrations of the appendage is amplified with time. The system may become unstable to an order of approximation used here. For case 2, the time-response characteristics of the system are shown in Fig. 6, where the system parameters are given by

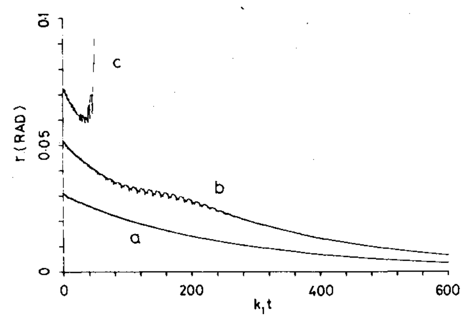

Fig. 5a Plot of r against time (Case 1).

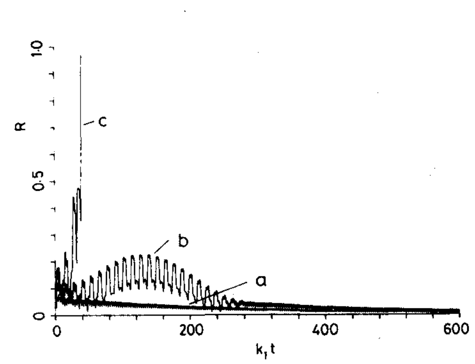

Fig. 5b Plot of R against time (Case 1).

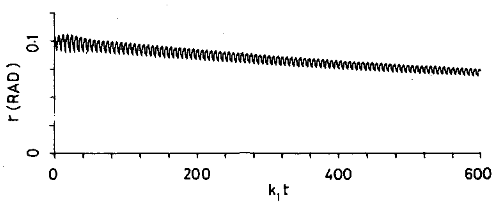

Fig. 6a Plot of r against time (Case 2).

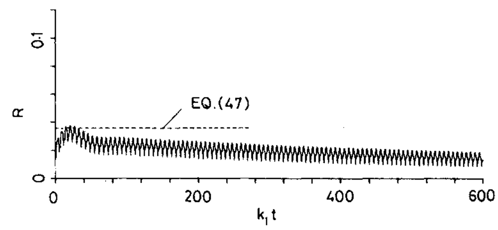

Fig. 6b Plot of R against time (Case 2).

$$\gamma/k_1=0.05, \quad k/(2k_1)^2\mu\ell^3=0.12, \quad \Delta_2/4k_1^2=0.15 \quad (50)$$

The initial value $a=0.1$ rad. is adopted for the amplitude of θ_2. The result shows that the amplitude of θ_2 gradually decreases in the course of time owing to the energy dissipation in the appendage. The vibration of the appendage, on the other hand, rapidly builds up to the value, which is predicted by Eq.(47), and gradually damps as the amplitude of θ_2 decreases.

Conclusion

In this paper, two types of thermally induced vibration have been discussed for a flexible appendage attached to a spacecraft containing a rotor with particular emphasis on the behavior near resonance. The first type of resonance arises when the natural frequency of the appendage is approximately equal to twice the nutational frequency. The second one arises when the natural frequency of the appendage is in the neighborhood of half the nutational frequency. These resonance phenomena can be considered as an autoparametric excitation of a mechanical system with several degrees of freedom. The method of averaging is used to investigate the conditions for the resonance, the stability condition for the first case [Eq. (40)], and the stationary amplitude of the vibration for the second case [Eq. (47)]. The time behaviors of the system also have been investigated numerically.

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